Corporate Finance

Net Present Value (NPV)

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - Outlay$$

where
CF_t = after-tax cash flow at time, t.
r = required rate of return for the investment. This is the firm's cost of capital adjusted for the risk inherent in the project.
Outlay = investment cash outflow at t = 0.

Internal Rate of Return (IRR)

$$\sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} = Outlay \qquad \sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} - Outlay = 0$$

Average Accounting Rate of Return (AAR)

$$AAR = \frac{\text{Average net income}}{\text{Average book value}}$$

Profitability Index

$$PI = \frac{\text{PV of future cash flows}}{\text{Initial investment}} = 1 + \frac{NPV}{\text{Initial investment}}$$

Weighted Average Cost of Capital

$$WACC = (w_d)(r_d)(1-t) + (w_p)(r_p) + (w_e)(r_e)$$

Where:
w_d = Proportion of debt that the company uses when it raises new funds
r_d = Before-tax marginal cost of debt
t = Company's marginal tax rate
w_p = Proportion of preferred stock that the company uses when it raises new funds
r_p = Marginal cost of preferred stock
w_e = Proportion of equity that the company uses when it raises new funds
r_e = Marginal cost of equity

To Transform Debt-to-equity Ratio into a component's weight

$$\frac{D/E}{1 + D/E} = \frac{D}{D+E} = w_d$$

$$w_d + w_e = 1$$

Valuation of Bonds

$$P_0 = \left[\sum_{t=1}^{n} \frac{PMT}{\left(1 + \frac{r_d}{2}\right)^t} \right] + \frac{FV}{\left(1 + \frac{r_d}{2}\right)^n}$$

where:
P_0 = current market price of the bond.
PMT_t = interest payment in period t.
r_d = yield to maturity on BEY basis.
n = number of periods remaining to maturity.
FV = Par or maturity value of the bond.

Valuation of Preferred Stock

$$V_p = D_p / r_p$$

where:
V_p = current value (price) of preferred stock..
D_p = preferred stock dividend per share.
r_p = cost of preferred stock.

Required Return on a Stock

Capital Asset Pricing Model

$$r_e = R_F + \beta_i[E(R_M) - R_F]$$

where
$[E(R_M) - R_F]$ = Equity risk premium.
R_M = Expected return on the market.
β_i = Beta of stock . Beta measures the sensitivity of the stock's returns to changes in market returns.
R_F = Risk-free rate.
r_e = Expected return on stock (cost of equity)

Dividend Discount Model

$$P_0 = \frac{D_1}{r_e - g}$$

where:
P_0 = current market value of the security.
D_1= next year's dividend.
r_e = required rate of return on common equity.
g = the firm's expected constant growth rate of dividends.

Rearranging the above equation gives us a formula to calculate the required return on equity:

$$r_e = \frac{D_1}{P_0} + g$$

Sustainable Growth Rate

$$g = \left(1 - \frac{D}{EPS}\right) \times (ROE)$$

Where (1 - (D/EPS)) = Earnings retention rate

Bond Yield plus Risk Premium Approach

$$r_e = r_d + \text{ risk premium}$$

To Unlever the beta

$$\beta_{ASSET} = \beta_{EQUITY}\left[\frac{1}{1 + \left((1-t)\frac{D}{E}\right)}\right]$$

To Lever the beta

$$\beta_{PROJECT} = \beta_{ASSET}\left[1 + \left((1-t)\frac{D}{E}\right)\right]$$

Country Risk Premium

$$r_e = R_F + \beta\ [E(R_M) - R_F + CRP]$$

$$\text{Country risk premium} = \text{Sovereign yield spread} \times \frac{\text{Annualized standard deviation of equity index}}{\text{Annualized standard deviation of sovereign bond market in terms of the developed market currency}}$$

$$\text{Break point} = \frac{\text{Amount of capital at which a component's cost of capital changes}}{\text{Proportion of new capital raised from the component}}$$

Degree of Operating Leverage

$$DOL = \frac{\text{Percentage change in operating income}}{\text{Percentage change in units sold}}$$

$$DOL = \frac{Q \times (P - V)}{Q \times (P - V) - F}$$

where:
Q = Number of units sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
Q × (P – V) = Contribution margin (the amount that units sold contribute to covering fixed costs)
(P – V) = Contribution margin per unit

Degree of Financial Leverage

$$DFL = \frac{\text{Percentage change in net income}}{\text{Percentage change in operating income}}$$

$$DFL = \frac{[Q(P - V) - F](1 - t)}{[Q(P - V) - F - C](1 - t)} = \frac{[Q(P - V) - F]}{[Q(P - V) - F - C]}$$

where:
Q = Number of units sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
C = Fixed financial cost
t = Tax rate

Degree of Total Leverage

$$DTL = \frac{\text{Percentage change in net income}}{\text{Percentage change in the number of units sold}}$$

$$DTL = DOL \times DFL$$

$$DTL = \frac{Q \times (P - V)}{[Q(P - V) - F - C]}$$

where:
Q = Number of units produced and sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
C = Fixed financial cost

Break point

$PQ = VQ + F + C$

where:
P = Price per unit
Q = Number of units produced and sold
V = Variable cost per unit
F = Fixed operating costs
C = Fixed financial cost

The breakeven number of units can be calculated as:

$$Q_{BE} = \frac{F + C}{P - V}$$

Operating breakeven point

$PQ_{OBE} = PV + F$

$$Q_{OBE} = \frac{F}{P - V}$$

$$\text{Current Ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{Quick Ratio} = \frac{\text{cash} + \text{short term marketable investments} + \text{receivables}}{\text{Current liabilities}}$$

$$\text{Accounts receivable turnover} = \frac{\text{Credit sales}}{\text{Average receivables}}$$

$$\text{Number of days of receivables} = \frac{\text{Accounts receivable}}{\text{Average day s sales on credit}}$$

$$= \frac{\text{Accounts receivable}}{\text{Sales on credit / 365}}$$

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Number of days of inventory} = \frac{\text{Inventory}}{\text{Average day's cost of goods sold}}$$

$$= \frac{\text{Inventory}}{\text{Cost of goods sold / 365}}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Average trade payables}}$$

$$\text{Number of days of payables} = \frac{\text{Accounts payables}}{\text{Average day's purchases}}$$

$$= \frac{\text{Accounts payables}}{\text{Purchases/365}}$$

Purchases = Ending inventory + COGS - Beginning inventory

Operating cycle = Number of days of inventory + Number of days of receivables

Net operating cycle = Number of days of inventory + Number of days of receivables
-Number of days of payables

$$\text{Money market yield} = \left(\frac{\text{Face value - price}}{\text{Price}}\right) \times \left(\frac{360}{\text{Days}}\right) = \text{Holding period yield} \times \left(\frac{360}{\text{Days}}\right)$$

$$\text{Bond equivalent yield} = \left(\frac{\text{Face value - price}}{\text{Price}}\right) \times \left(\frac{365}{\text{Days}}\right) = \text{Holding period yield} \times \left(\frac{365}{\text{Days}}\right)$$

$$\text{Discount basis yield} = \left(\frac{\text{Face value - price}}{\text{Face value}}\right) \times \left(\frac{360}{\text{Days}}\right) = \%\ \text{discount} \times \left(\frac{360}{\text{Days}}\right)$$

$$\% \text{ Discount} = \frac{\text{Face value - Price}}{\text{Price}}$$

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Number of days of inventory} = \frac{\text{Inventory}}{\text{Average days cost of goods sold}}$$

$$= \frac{\text{Inventory}}{\text{Cost of goods sold / 365}}$$

$$= \frac{365}{\text{Inventory turnover}}$$

$$\text{Implicit rate} = \text{Cost of trade credit} = \left(1 + \frac{\text{Discount}}{1 - \text{Discount}}\right)^{\left(365 \big/ \text{Number of days beyond discount period}\right)} - 1$$

$$\text{Number of days of payables} = \frac{\text{Accounts payable}}{\text{Average day's purchases}}$$

$$\frac{\text{Accounts payable}}{\text{Purchases / 365}} = \frac{365}{\text{Payables turnover}}$$

$$\text{Line of credit cost} = \frac{\text{Interest} + \text{Commitment fee}}{\text{Loan amount}}$$

$$\text{Banker's acceptance cost} = \frac{\text{Interest}}{\text{Net proceeds}} = \frac{\text{Interest}}{\text{Loan amount - Interest}}$$

$$\frac{\text{Interest} + \text{Dealer's comission} + \text{Backup costs}}{\text{Loan amount - Interest}}$$

$$ROE = \frac{\text{Net income}}{\text{Average total equity}}$$

$$ROE = \frac{\text{Net income}}{\text{Average total assets}} \times \frac{\text{Average total assets}}{\text{Average shareholders' equity}}$$

$$ROE = \frac{\text{Net income}}{\text{Revenue}} \times \frac{\text{Revenue}}{\text{Average total assets}} \times \frac{\text{Average total assets}}{\text{Average shareholders' equity}}$$

Portfolio Management

Holding Period Return

$$R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}} = \text{Capital gain} + \text{Dividend yield}$$

$$= \frac{P_T + D_T}{P_0} - 1$$

where:
P_t = Price at the end of the period
P_{t-1} = Price at the beginning of the period
D_t = Dividend for the period

Holding Period Returns for more than One Period

$$R = [(1 + R_1) \times (1 + R_2) \times \ldots\times (1 + R_n)] – 1$$

where:
$R_1, R_2, \ldots, R_n$ are sub-period returns

Geometric Mean Return

$$R = \{[(1 + R_1) \times (1 + R_2) \times \ldots\times (1 + R_n)]^{1/n}\} – 1$$

Annualized Return

$$r_{annual} = (1 + r_{period})^n - 1$$

where:
r = Return on investment
n = Number of periods in a year

Portfolio Return

$R_p = w_1R_1 + w_2R_2$

where:
R_p = Portfolio return
w_1 = Weight of Asset 1
w_2 = Weight of Asset 2
R_1 = Return of Asset 1
R_2 = Return of Asset 2

Variance of a Single Asset

$$\sigma^2 = \frac{\sum_{t=1}^{T} (R_t - \mu)^2}{T}$$

where:
R_t = Return for the period t
T = Total number of periods
μ = Mean of T returns

Variance of a Representative Sample of the Population

$$s^2 = \frac{\sum_{t=1}^{T} (R_t - \bar{R})^2}{T-1}$$

where:
R = mean return of the sample observations
s^2 = sample variance

Standard Deviation of an Asset

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T} (R_t - \mu)^2}{T}} \qquad s = \sqrt{\frac{\sum_{t=1}^{T} (R_t - \bar{R})^2}{T-1}}$$

Variance of a Portfolio of Assets

$$\sigma_P^2 = \sum_{i,j=1}^{N} w_i w_j \mathrm{Cov}(R_i, R_j)$$

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \mathrm{Var}(R_i) + \sum_{i,j=1,\, i \neq j}^{N} w_i w_j \mathrm{Cov}(R_i, R_j)$$

Standard Deviation of a Portfolio of Two Risky Assets

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}} \text{ or } \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\mathrm{Cov}_{1,2}}$$

Utility Function

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:
U = Utility of an investment
E(R) = Expected return
σ^2 = Variance of returns
A = Additional return required by the investor to accept an additional unit of risk.

Capital Allocation Line

The CAL has an intercept of RFR and a constant slope that equals:

$$\frac{[E(R_i) - RFR]}{\sigma_i}$$

Expected Return on portfolios that lie on CML

$$E(R_p) = w_1R_f + (1 - w_1)\,E(R_m)$$

Variance of portfolios that lie on CML

$$\sigma^2 = \sqrt{w_1^2\sigma_f^2 + (1 - w_1)^2\sigma_m^2 + 2w_1(1 - w_1)\mathrm{Cov}(R_f,R_m)}$$

Equation of CML

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m} \times \sigma_p$$

where:
y-intercept = R_f = risk-free rate
slope = $\frac{E(R_m) - R_f}{\sigma_m}$ = market price of risk.

Systematic and Nonsystematic Risk

Total Risk = Systematic risk + Unsystematic risk

Return-Generating Models

$$E(R_i) - R_f = \sum_{j=1}^{k} \beta_{ij} E(F_j) = \beta_{i1}[E(R_m) - R_f] + \sum_{j=2}^{k} \beta_{ij} E(F_j)$$

The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Calculation of Beta

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

The Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Sharpe ratio

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

Treynor ratio

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p}$$

M-squared (M^2)

$$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} - (R_m - R_f)$$

Jensen's alpha

$$\alpha_p = R_p - [R_f + \beta_p (R_m - R_f)]$$

Security Characteristic Line

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f)$$

Equity

The price at which an investor who goes long on a stock receives a margin call is calculated as:

$$P_0 \times \frac{(1 - \text{Initial margin})}{(1 - \text{Maintenance margin})}$$

The value of a price return index is calculated as follows:

$$V_{PRI} = \frac{\sum_{i=1}^{N} n_i P_i}{D}$$

where:
V_{PRI} = Value of the price return index
n_i = Number of units of constituent security i held in the index portfolio
N = Number of constituent securities in the index
P_i = Unit price of constituent security i
D = Value of the divisor

Price Return

The price return of an index can be calculated as:

$$PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

where:
PR_I = Price return of the index portfolio (as a decimal number)
V_{PRI1} = Value of the price return index at the end of the period
V_{PRI0} = Value of the price return index at the beginning of the period

The price return of each constituent security is calculated as:

$$PR_i = \frac{P_{i1} - P_{i0}}{P_{i0}}$$

where:
PR_i = Price return of constituent security i (as a decimal number)
P_{i1} = Price of the constituent security i at the end of the period
P_{i0} = Price of the constituent security i at the beginning of the period

The price return of the index equals the weighted average price return of the constituent securities. It is calculated as:

$$PR_I = w_1 PR_1 + w_2 PR_2 + ...+ w_N PR_N$$

where:
PR_I = Price return of the index portfolio (as a decimal number)
PR_i = Price return of constituent security i (as a decimal number)
w_i = Weight of security i in the index portfolio
N = Number of securities in the index

Total Return

The total return of an index can be calculated as:

$$TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:
TR_I = Total return of the index portfolio (as a decimal number)
V_{PRI1} = Value of the total return index at the end of the period
V_{PRI0} = Value of the total return index at the beginning of the period
Inc_I = Total income from all securities in the index held over the period

The total return of each constituent security is calculated as:

$$TR_i = \frac{P_{1i} - P_{0i} + Inc_i}{P_{0i}}$$

where:
TR_i = Total return of constituent security i (as a decimal number)
P_{1i} = Price of constituent security i at the end of the period
P_{0i} = Price of constituent security i at the beginning of the period
Inc_i = Total income from security i over the period

The total return of the index equals the weighted average total return of the constituent securities. It is calculated as:

$$TR_I = w_1 TR_1 + w_2 TR_2 + ...+ w_N TR_N$$

where:
TR_I = Total return of the index portfolio (as a decimal number)
TR_i = Total return of constituent security i (as a decimal number)
w_i = Weight of security i in the index portfolio
N = Number of securities in the index

Calculation of Index Returns over Multiple Time Periods

Given a series of price returns for an index, the value of a price return index can be calculated as:

$$V_{PRIT} = V_{PRI0}\,(1 + PR_{I1})\,(1 + PR_{I2})\,...\,(1 + PR_{IT})$$

where:
V_{PRI0} = Value of the price return index at inception
V_{PRIT} = Value of the price return index at time t
PR_{IT} = Price return (as a decimal number) on the index over the period

Similarly, the value of a total return index may be calculated as:

$$V_{TRIT} = V_{TRI0}\,(1 + TR_{I1})\,(1 + TR_{I2})\,...\,(1 + TR_{IT})$$

where:
V_{TRI0} = Value of the index at inception
V_{TRIT} = Value of the index at time t
TR_{IT} = Total return (as a decimal number) on the index over the period

Price Weighting

$$w_i^P = \frac{P_i}{\sum_{i=1}^{N} P_i}$$

Equal Weighting

$$w_i^E = \frac{1}{N}$$

where:
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
N = Number of securities in the index

Market-Capitalization Weighting

$$w_i^M = \frac{Q_i P_i}{\sum_{j=1}^{N} Q_j P_j}$$

where:
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
Q_i = Number of shares outstanding of security i
P_i = Share price of security i
N = Number of securities in the index

The float-adjusted market-capitalization weight of each constituent security is calculated as:

$$w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^{N} f_j Q_j P_j}$$

where:
f_i = Fraction of shares outstanding in the market float
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
Q_i = Number of shares outstanding of security i
P_i = Share price of security i
N = Number of securities in the index

Fundamental Weighting

$$w_i^F = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

where:
F_i = A given fundamental size measure of company i

Return Characteristics of Equity Securities

Total Return, $R_t = (P_t - P_{t-1} + D_t) / P_{t-1}$

where:
P_{t-1} = Purchase price at time t – 1
P_t = Selling price at time t
D_t = Dividends paid by the company during the period

Accounting Return on Equity

$$ROE_t = \frac{NI_t}{\text{Average } BVE_t} = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Dividend Discount Model (DDM)

$$\text{Value} = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \ldots + \frac{D_\infty}{(1+k_e)^\infty}$$

$$\text{Value} = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t}$$

One year holding period:

$$\text{Value} = \frac{\text{dividend to be received}}{(1+k_c)^1} + \frac{\text{year-end price}}{(1+k_c)^1}$$

Multiple-Year Holding Period DDM

$$V = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \dots + \frac{P_n}{(1+k_e)^n}$$

where:
P_n = Price at the end of n years.

Infinite Period DDM (Gordon Growth Model)

$$PV_0 = \frac{D_0\,(1+g_c)^1}{(1+k_e)^1} + \frac{D_0\,(1+g_c)^2}{(1+k_e)^2} + \frac{D_0\,(1+g_c)^3}{(1+k_e)^3} + \dots + \frac{D_0\,(1+g_c)^\infty}{(1+k_e)^\infty}$$

This equation simplifies to:

$$PV = \frac{D_0\,(1+g_c)^1}{(k_e - g_c)^1} = \frac{D_1}{k_e - g_c}$$

The long-term (constant) growth rate is usually calculated as:

$g_c = RR \times ROE$

Multi-Stage Dividend Discount Model

$$\text{Value} = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \dots + \frac{D_n}{(1+k_e)^n} + \frac{P_n}{(1+k_e)^n}$$

where:

$$P_n = \frac{D_{n+1}}{k_c - g_c}$$

D_n = Last dividend of the supernormal growth period
D_{n+1} = First dividend of the constant growth period

The Free-Cash-Flow-to-Equity (FCFE) Model

FCFE = CFO – FC Inv + Net borrowing

Analysts may calculate the intrinsic value of the company's stock by discounting their projections of future FCFE at the required rate of return on equity.

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1 + k_e)^t}$$

Value of a Preferred Stock

When preferred stock is non-callable, non-convertible, has no maturity date and pays dividends at a fixed rate, the value of the preferred stock can be calculated using the perpetuity formula:

$$V_0 = \frac{D_0}{r}$$

For a non-callable, non-convertible preferred stock with maturity at time, n, the value of the stock can be calculated using the following formula:

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1 + r)^t} + \frac{F}{(1 + r)^n}$$

where:
V_0 = value of preferred stock today (t = 0)
D_t = expected dividend in year t, assumed to be paid at the end of the year
r = required rate of return on the stock
F = par value of preferred stock

Price Multiples

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{r - g}$$

$$\text{Price to cash flow ratio} = \frac{\text{Market price of share}}{\text{Cash flow per share}}$$

$$\text{Price to sales ratio} = \frac{\text{Market price per share}}{\text{Net sales per share}}$$

$$\text{Price to sales ratio} = \frac{\text{Market value of equity}}{\text{Total net sales}}$$

$$P/BV = \frac{\text{Current market price of share}}{\text{Book value per share}}$$

$$P/BV = \frac{\text{Market value of common shareholders' equity}}{\text{Book value of common shareholders' equity}}$$

where:
Book value of common shareholders' equity =
(Total assets - Total liabilities) - Preferred stock

Enterprise Value Multiples

EV/EBITDA

where:

EV = Enterprise value and is calculated as the market value of the company's common stock plus the market value of outstanding preferred stock if any, plus the market value of debt, less cash and short term investments (cash equivalents).

Fixed Income

Bond Coupon

Coupon = Coupon rate × Par value

Coupon Rate (Floating)

Coupon Rate = Reference rate + Quoted margin

Coupon Rate (Inverse Floaters)

Coupon rate = K – L× (Reference rate)

Callable Bond Price

Price of a callable bond = Value of option-free bond – Value of embedded call option

Putable Bond Price

Price of a putable bond = Value of option-free bond + Value of embedded put option

Dollar Duration

Dollar duration = Duration × Bond value

Inflation-Indexed Treasury Securities

TIPS coupon = Inflation adjusted par value × (Stated coupon rate/2)

Nominal spread

Nominal spread (Bond Y as the reference bond) = Yield on Bond X – Yield on Bond Y

Relative Yield spread

$$\text{Relative yield spread} = \frac{\text{Yield on Bond X} - \text{Yield on Bond Y}}{\text{Yield on Bond Y}}$$

Yield Ratio

$$\text{Yield ratio} = \frac{\text{Yield on Bond X}}{\text{Yield on Bond Y}}$$

After-Tax Yield

After-tax yield = Pretax yield × (1- marginal tax rate)

Taxable-Equalent Yield

$$\text{Taxable-equivalent yield} = \frac{\text{Tax-exempt yield}}{(1 - \text{marginal tax rate})}$$

Bond Value

$$\text{Bond Value} = \frac{\text{Maturity value}}{(1+i)^{\text{years till maturity} \times 2}}$$

where i equals the semiannual discount rate

Valuing a Bond Between Coupon Payments.

$$w = \frac{\text{Days between settlement date and next coupon payment date}}{\text{Days in coupon period}}$$

where:
w = Fractional period between the settlement date and the next coupon payment date.

$$\text{Present value}_t = \frac{\text{Expected cash flow}}{(1+i)^{t-1+w}}$$

Current Yield

$$\text{Current yield} = \frac{\text{Annual cash coupon}}{\text{Bond price}}$$

Bond Price

$$\text{Bond price} = \frac{CPN_1}{\left(1+\frac{YTM}{2}\right)} + \frac{CPN_2}{\left(1+\frac{YTM}{2}\right)^2} + \frac{CPN_{2N} + Par}{\left(1+\frac{YTM}{2}\right)^{2N}}$$

where:
Bond price = Full price including accrued interest.
CPN_t = The semiannual coupon payment received after t semiannual periods.
N = Number of years to maturity.
YTM = Yield to maturity.

Formula to Convert BEY into Annual-Pay YTM:

$$\text{Annual-pay yield} = \left[\left(1 + \frac{\text{Yield on bond equivalent basis}}{2}\right)^2 - 1\right]$$

Formula to Convert Monthly Cash Flow Yield into BEY

$$BEY = [(1 + \text{monthly CFY})^6 - 1] \times 2$$

Discount Basis Yeild

$$d = (1-p)\frac{360}{N}$$

Z-Spread

Z-spread = OAS + Option cost; and OAS = Z-spread - Option cost

Duration

$$\text{Duration} = \frac{V_- - V_+}{2(V_0)(\Delta y)}$$

where:
Δy = change in yield in decimal
V_0 = initial price
V_- = price if yields decline by Δy
V_+ = price if yields increase by Δy

Portfolio Duration

Portfolio duration = $w_1D_1 + w_2D_2 + \ldots + w_ND_N$

where:
N = Number of bonds in portfolio.
D_i = Duration of Bond i.
w_i = Market value of Bond i divided by the market value of portfolio.

Percentage Change in Bond Price

Percentage change in bond price = duration effect + convexity adjustment
= $\{[-\text{duration} \times (\Delta y)] + [\text{convexity} \times (\Delta y)^2]\} \times 100$

where:
Δy = Change in yields in decimals.

Convexity

$$C = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

Price Value of a Basis Point

Price value of a basis point = Duration × 0.0001 × bond value

DERIVATIVES

FRA Payoff

$$\frac{\text{Floating rate at expiration} - \text{FRA rate} \times (\text{days in floating rate}/\ 360)}{1 + [\text{Floating rate at expiration} \times (\text{days in floating rate}/\ 360)}$$

Numerator: Interest savings on the hypothetical loan. This number is positive when the floating rate is greater than the forward rate. When this is the case, the long benefits and expects to receive a payment from the short. The numerator is negative when the floating rate is lower than the forward rate. When this is the case, the short benefits and expects to receive a payment from the long.

Denominator: The discount factor for calculating the present value of the interest savings.

Call Option Payoffs

Option Position	Description	Payoffs	
		$S_T > X$	$S_T < X$
		Option holder exercises the option.	Option holder does not exercise the option.
Call option holder	Choice to buy the underlying asset for X	$S_T - X$	0
Call option writer	Obligation to sell the underlying asset for X if the option holder chooses to exercise the option	$-(S_T - X)$	0

Intrinsic Value of a Call Option

Intrinsic value of call = Max [0, (S_t - X)]

Put Option Payoffs

Option Position	Description	Payoffs	
		$S_T < X$	$S_T > X$
		Option holder exercises the option	Option holder does not exercise the option
Put option holder	Choice to sell the underlying asset for X	$X - S_T$	0
Put option writer	Obligation to buy the underlying asset for X if the option holder chooses to exercise the option	$-(X - S_T)$	0

Moneyness and Intrinsic Value of a Put Option

Moneyness	Current Market Price (S_t) versus Exercise Price (X)	Intrinsic Value Max [0, (X - S_t)]
In-the-money	S_t is less than X	$X – S_t$
At-the-money	S_t equals X	0
Out-of-the-money	S_t is greater than X	0

Option Premium

Option premium = Intrinsic value + Time value

Put-Call Parity

$$C_0 + \frac{X}{(1 + R_F)^T} = P_0 + S_0$$

Synthetic Derivative Securities

Strategy	Consisting of	Value	Equals	Strategy	Consisting of	Value
fiduciary call	long call + long bond	$C_0 + \frac{X}{(1 + R_F)^T}$	=	Protective put	long put + long underlying asset	$P_0 + S_0$
long call	long call	C_0	=	Synthetic call	long put + long underlying asset + short bond	$P_0 + S_0 - X/(1+R_F)^T$
long put	long put	P_0	=	Synthetic put	long call + short underlying asset + long bond	$C_0 - S_0 + X/(1+R_F)^T$
long underlying asset	long underlying asset	S_0	=	Synthetic underlying asset	long call + long bond + short put	$C_0 + X/(1+R_F)^T - P_0$
long bond	long bond	$\frac{X}{(1 + R_F)^T}$	=	Synthetic bond	long put + long underlying asset + short call	$P_0 + S_0 - C_0$

Option Value Limits

Option	Minimum Value	Maximum Value
European call	$EC_t \geq 0$	$EC_t \leq S_t$
American call	$AC_t \geq 0$	$AC_t \leq S_t$
European put	$EP_t \geq 0$	$EP_t \leq X/(1 + RFR)^T$
American put	$AP_t \geq 0$	$AP_t \leq X$

Option Value Bounds

Option	Minimum Value	Maximum Value
European Call	$\text{Max}\left[0, S_t - \frac{X}{(1+RFR)^T}\right]$	S_t
American Call	$\text{Max}\left[0, S_t - \frac{X}{(1+RFR)^T}\right]$	S_t
European Put	$\text{Max}\left[0, \frac{X}{(1+RFR)^T} - S_t\right]$	$\frac{X}{(1+RFR)^T}$
American Put	$\text{Max}\left[0, X - S_t\right]$	X

Interest Rate Call Holder's Payoff

$$= \text{Max}(0, \text{Underlying rate at expiration} - \text{Exercise rate}) \frac{(\text{Days in underlying Rate})}{360} \times NP$$

where: NP = Notional principal

Interest Rate Put Holder's Payoff

$$= \text{Max}(0, \text{Exercise rate} - \text{Underlying rate at expiration}) \frac{(\text{Days in underlying rate})}{360} \times NP$$

where:
NP = Notional principal

Net Payment for a Fixed-Rate-Payer

Net fixed-rate payment$_t$ = (Swap fixed rate - LIBOR$_{t-1}$)×(No. of days/360)×(NP)
where:
NP equals the notional principal.

Summary of Options Strategies

	Call	**Put**
Holder	$C_T = \max(0,S_T - X)$ Value at expiration = C_T Profit: $\Pi = C_T - C_0$ Maximum profit = ∞ Maximum loss = C_0 Breakeven: $S_T^* = X + C_0$	$P_T = \max(0,X - S_T)$ Value at expiration = P_T Profit: $\Pi = P_T - P_0$ Maximum profit = $X - P_0$ Maximum loss = P_0 Breakeven: $S_T^* = X - P_0$
Writer	$C_T = \max(0,S_T-X)$ Value at expiration = $-C_T$ Profit: $\Pi = -C_T - C_0$ Maximum profit = C_0 Maximum loss = ∞ Breakeven: $S_T^* = X + C_0$	$P_T = \max(0,X - S_T)$ Value at expiration = $-P_T$ Profit: $\Pi = -P_T - P_0$ Maximum profit = P_0 Maximum loss = $X - P_0$ Breakeven: $S_T^* = X - P_0$

Where:
- C_0, C_T = price of the call option at time 0 and time T
- P_0, P_T = price of the put option at time 0 and time T
- X = exercise price
- S_0, S_T = price of the underlying at time 0 and time T
- V_0, V_T = value of the position at time 0 and time T
- Π = profit from the transaction: $V_T - V_0$
- r = risk-free rate

Covered Call

Value at expiration: $V_T = S_T - \max(0,S_T - X)$
Profit: $\Pi = V_T - S_0 + C_0$
Maximum profit = $X - S_0 + C_0$
Maximum loss = $S_0 - C_0$
Breakeven: $S_T^* = S_0 - C_0$

Protective Put

Value at expiration: $V_T = S_T + \max(0,X - S_T)$
Profit: $\Pi = V_T - S_0 - P_0$
Maximum profit = ∞
Maximum loss = $S_0 + P_0 - X$
Breakeven: $S_T^* = S_0 + P_0$