2013, Study Session # 3, Reading # 11

"HYPOTHESIS TESTING"

$\propto$ = Level of Significance
TS = Test Statistics
TV = Table Value

SS = Sample Statistic
CV = Critical Value
SE = Standard Error

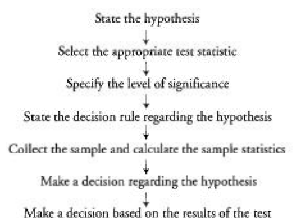
Hypothesis
Statement about parameter value developed for testing.

Two Types

Null Hypothesis H_0
- Tested for possible rejection.
- Always includes '=' sign.

Alternative Hypothesis H_a
The one that we want to prove.

Hypothesis Testing Procedure
- It is based on sample statistics & probability theory.
- It is used to determine whether a hypothesis is a reasonable statement or not.



(*Source:* Wayne W. Daniel and James C. Terrell, *Business Statistics, Basic Concepts and Methodology,* Houghton Mifflin, Boston, 1997.)

One Tailed Test
Alternative hypothesis having one side.
- Upper Tail
  $H_0: \mu \le \mu_0$ vs $H_a: \mu > \mu_0$.
- Decision rule
  Reject H_0 if TS > TV
- Lower Tail
  $H_0: \mu \ge \mu_0$ vs $H_a: \mu < \mu_0$.
- Decision rule
  Reject H_0 if TS < - TV

Two Tailed Test
- Alternative hypothesis having two sides.
- $H_0: \mu = \mu_0$ vs $H_a\ \mu \ne \mu_0$.
- Reject H_0 if
  $|TS| > TV$

Test Statistics
Hypothesis testing involves two statistics:
- TS calculated from sample data.
- critical values of TS.

TS is a random variable that follows some distribution.

Two Types of Errors

Type I Error
Rejecting a true null hypothesis.

Type II Error
Failing to reject a false null hypothesis.

Decision Rule
- It is based on distribution of TS.
- It is specific & quantitative.

Significance Level (α)
- Probability of making a type I error.
- Denoted by Greek letter alpha (α).
- Used to identify critical values.

Statistical Significance vs Economical Significance
- Statistically significant results may not necessarily be economically significant.
- A very large sample size may result in highly statistically significant results that may be quite small in absolute terms.

σ^2 = Population Variance
N.Dist = Normally Distributed
N.N.Dist = Non Normally

n = Sample Size

df = Degree of Freedom
n ≥ 30 = Large Sample
n< 30 = Small Sample

Relationship b/w Confidence Intervals & Hypothesis Tests

- Related because of critical value.

C.I

- [(SS)- (CV)(SE)] ≤ parameter ≤ [(SS) + (CV)(SE)].
- It gives the range within which parameter value is believed to lie given a level of confidence.

Hypothesis Test

- -C V ≤ TS ≤ + CV.
- range within which we fail to reject null hypothesis of two tailed test given level of significance.

p- value

- Probability of obtaining a critical value that would lead to a rejection of a true null hypothesis.
- Reject H_0 if p-value < α.

Power of a Test

- P(type II error).
- Probability of correctly rejecting a false null hypothesis.

Testing	Conditions	Test Statistics	Decision Rule
Population Mean	• σ^2 known • N. dist.	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	• H_o:$\mu \leq \mu_0$ vs H_a: $\mu > \mu_0$ Reject H_0 if TS. > TV • H_o:$\mu \geq \mu_0$ vs H_a: $\mu < \mu_0$ Reject H_0 if TS. <TV • H_o:$\mu = \mu_0$ vs H_a: $\mu \neq \mu_0$ Reject H_0 if | TS | > TV
	• n ≥30 • σ^2 unknown	$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ or $t^*_{n-1} = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ *(more conservative)	
	• σ^2 unknown • n<30 • N. dist.	$t_{n-1} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$; df = n-1	
Equality of the Means of Two Normally Distributed Populations based on Independent Samples.	Unknown variances assumed equal.	$t_{(n_1+n_2-2)} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_P\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ where; $s_P = \sqrt{\frac{(n_1 - 1)s_1{}^2 + (n_2 - 1)s_2{}^2}{n_1 + n_2 - 2}}$ df = n_1+n_2 - 2	• H_o:μ_1 - $\mu_2 \leq 0$ vs Ha: μ_1 -μ_2 > 0 Reject H_0 if TS > TV • H_o:μ_1 - $\mu_2 \geq 0$ vs Ha: μ_1 -μ_2 < 0 Reject H_0 if TS < -TV • H_o:μ_1 - μ_2 = 0 vs Ha: μ_1 - $\mu_2 \neq 0$ Reject H_0 if | TS | > TV
	Unequal unknown variances.	$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$	

$$d.f = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2}}$$

Paired Comparisons Test

TS $t_{(n-1)} = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}}$

$$\bar{d} = \frac{1}{n} . \Sigma d$$

$$S_{\bar{d}} = \frac{S_d}{\sqrt{n}}$$

$$s_d = \sqrt{\frac{\Sigma (d - \bar{d})^2}{n-1}}$$

Decision Rule

- $H_0: \mu_d \leq \mu_{d0}$ vs $H_a: \mu_d > \mu d_0$
  Reject H_0 if TS > TV.
- $H_0: \mu_d \geq \mu_{d0}$ vs $Ha: \mu_d < \mu_{d0}$
  Reject H_0 if TS <-TV
- $H_0: \mu_d = \mu_{d0}$ vs $H_a: \mu d \neq \mu_{d0}$
  Reject H_0 if TS > TV.

Testing Variance of a N.dist. Population

TS

$$\chi^2_{(n-1)} = \frac{(n-1)s^2}{\sigma_0^2}$$

Decision Rule

Reject H_0 if TS > TS

Chi-Square Distribution

- Asymmetrical.
- Bounded from below by zero.
- Chi-Square values can never be –ve.

Testing Equality of Two Variances from N.dist. Population

TS

$$F = \frac{S_1^2}{S_2^2} \; ; \; S_1^2 > S_2^2$$

Decision Rule

Reject H_0 if TS > TV

F- Distribution

- Right skewed.
- Bounded by zero.

Parametric Test

- Specific to population parameter.
- Relies on assumptions regarding the distribution of the population.

Non-Parametric Test

- Don't consider a particular population parameter.

Or

- Have few assumptions regarding population.