2013, Study Session # 3, Reading # 9

"COMMON PROBABILITY DISTRIBUTIONS"

Probability Distribution

- Describes the probabilities of all possible outcomes for a random variable.
- Sum of probabilities of all possible outcomes is 1.

Probability Function

Probability of a random variable being equal to a specific value.

Properties:

- $0 \leq p(x) \leq 1$
- $\Sigma\, p(x) = 1$

	Discrete	Continuous
Random Variable	Finite (measurable) # of possible outcomes.	Infinite (immeasurable) # of possible outcomes.
Distribution	• P(x) can't be 0 if 'x' can occur. • We can find the probability of a specific point in time.	• P(x) can be zero even if 'x' can occur. • We can't find the probability of a specific point in time.

Probability Density Function (PDF)

- It is used for continuous distribution.
- Denoted by f(x).

Cumulative Distribution Function (CDF)

- Calculates the probability of a random variable 'x' taking on the value less than or equal to a specific value of 'x'.
- $F(x) = P(X \leq x)$

Discrete uniform random variable ⇒ All outcomes have the same probability.

Uniform Probability Distribution

Discrete

- Has a finite number of specified outcomes.
- $P(x) \times k$. K is the probability for 'k' number of possible outcomes in a range.
- cdf: $F(x_n) = n.p(x)$.

Continuous

- Defined over a range with parameters 'b' (upper limit) & 'a' (lower limit).
- cdf: It is linear over the variable's range.
- Properties:
- $P(x \leq a) = 0$ & $P(x \geq b) = 1$
- $P(a < x < b) = \frac{x_2 - x_1}{b - a}$

Binomial Distribution

Properties:

- Two outcomes (success & failure).
- 'n' number of independent trials.
- Probability of success remains constant.
- p(x) =

$$\frac{n!}{(n-x)!.\,x!}\, p^x (1-p)^{n-x}$$

Binomial Tree

- Shows all possible combinations of up & down moves over a number of successive periods.
- Node: Each of the possible values along the tree.
- U is up-move factor.
- D is down-move factor (1/U).
- p is probability of up move.
- (1-p) is probability of down move.

Confidence Interval

Range of values around the expected value within which actual outcome is expected to be some specified percentage of time.

Confidence Interval	%age
$x \pm 1s$	68%
$x \pm 1.65s$	90%
$x \pm 1.96s$	95%
$x \pm 2s$	95.45%
$x \pm 2.58s$	99%
$x \pm 3s$	99.73%

Roy's Safety First Criterion

- Optimal portfolio minimizes the probability that the return of the portfolio falls below some minimum acceptable level.
- Minimize $P(R_P < R_L)$.
- SFRatio =

$$\frac{[E(R_P) - R_L]}{\sigma_P}$$

- Choose the portfolio with greatest SFRatio.

Shortfall Risk

Risk that portfolio value will fall below some minimum level at a future date.

Compounded Rates of Returns

Discrete

Daily, annually, weekly, monthly compounding

Continuous

- $\ln(S_1/S_0) = \ln(1+HPR)$
- These are additive for multiple periods.
- Effective annual rate based on continuous compounding is given as: $EAR = e^{Rcc} - 1$

Monte-Carlo Simulation

- Repeated generation of one or more factors (e.g. risk) that affect required value (e.g., stock price) in order to generate a distribution of the values (stock price).
- We have the flexibility of providing the data.

Simulation Procedure for Stock Option Valuation

Specify prob. dist. of stock prices & relevant interest rate as well as their parameters. → Randomly generate values of stock prices & interest rates. → Value the options for each pair of risk factors. → Calculate mean option value performing many iterations & use it as estimated option value.

Uses

- Valuing complex securities.
- Simulating gains / losses from trading strategy.
- Estimating value at risk (VAR).
- Examining variability of the difference b/w assets & liabilities of pension funds.
- Valuing portfolio with non - normal return distribution.

Limitations

- Complex procedure.
- Highly dependent on assumed distributions.
- Based on a statistical rather than an analytical method.

Historical Simulation

- Based on actual values & actual distribution of the factors i.e., based on historical data.

Limitation:

- History does not repeat itself.
- Historical data does not provide flexibility.